**PreCalculus 10.3 Ellipses**

**Ellipse -** the set of all points in a plane where the sum of the distances from two fixed points (foci) is a constant.

major axis = 2*a*

major axis = 2*a*

*F*1

*F*2

*F*1

*F*2

minor axis = 2*b*

minor axis = 2*b*

**Standard Form of the Equation of an Ellipse** with center $(h, k)$

 $\frac{\left(x-h\right)^{2}}{a^{2}}+\frac{\left(y-k\right)^{2}}{b^{2}}=1$ major axis is horizontal

 Foci are *c* units from center

 where $c^{2}=a^{2}-b^{2}$

 $\frac{\left(x-h\right)^{2}}{b^{2}}+\frac{\left(y-k\right)^{2}}{a^{2}}=1$ major axis is vertical

Example: Find the standard from of the equation of the ellipse.

 Center: $\left(-3, 4\right)$ $\frac{\left(x-h\right)^{2}}{b^{2}}+\frac{\left(y-k\right)^{2}}{a^{2}}=1$

$$\left(-3, 10\right)$$

 $a=6$ $\frac{\left(x--2\right)^{2}}{2^{2}}+\frac{\left(y-4\right)^{2}}{6^{2}}=1$

 $b=2$

 $\frac{\left(x+2\right)^{2}}{4}+\frac{\left(y-4\right)^{2}}{36}=1$

$$\left(-5, 4\right)$$

$$\left(-1, 4\right)$$

$$\left(-3, -2\right)$$

Example: Find the equation of the ellipse with foci $\left(-2, 2\right)$ and $\left(4, 2\right)$ with major axis = 10.

**Sketching an Ellipse**

Example: $\frac{\left(x-3\right)^{2}}{16}+\frac{\left(y+1\right)^{2}}{25}=1$

Example: Write the equation of the ellipse: $9x^{2}+4y^{2}+36x-8y+4=0$ in Standard Form.

 $9x^{2}+4y^{2}+36x-8y+4=0$

 $\left(9x^{2}+36x \right)+\left(4y^{2}-8y \right)=-4$

 $9\left(x^{2}+4x \right)+4\left(y^{2}-2y \right)=-4$

 $9\left(x^{2}+4x+4\right)+4\left(y^{2}-2y+1\right)=-4+36+4$

 $9\left(x+2\right)^{2} + 4\left(y-1\right)^{2} =36$

 $\frac{\left(x+2\right)^{2}}{4}+\frac{\left(y-1\right)^{2}}{9}=1$

Foci?

 $c^{2}=a^{2}-b^{2}=$

Example: Find the center, vertices and foci of the ellipse.

 $16x^{2}+25y^{2}-32x-50y+16=0$

When $a=b$, the ellipse is a circle

**Definition of Eccentricity:** The eccentricity of an ellipse is: $e=\frac{c}{a}$ $0<c<1$