**Chapter 3 – Exponential and Logarithmic Functions**

**PreCalculus 3.1 Exponential Functions and their Graphs**

**Exponential Function**

 $f\left(x\right)=a^{x}$ where $a>0$, $a\ne 1$

Example: Use a calculator to evaluate each function at the indicated value of *x*.

 $f\left(x\right)=8^{x}$ $x=π$

 $f\left(x\right)=8^{x}$ $x=\frac{1}{2}$

 $f\left(x\right)=8^{x}$ $x=-2.5$

**Graphs of Exponential Functions**

 $y=a^{x}$ points of the graph $\left(0, 1\right) and \left(1, a\right)$

 $\left(a>1\right)$ this is an increasing function

 *x*-axis is a horizontal asymptote

$$\left(1, a\right)$$

$$\left(0, 1\right)$$

 Domain: all real numbers

 Range: $y>0$

 This is a continuous function

 $y=a^{x}$ points of the graph $\left(0, 1\right) and \left(1, a\right)$

$$\left(0,1\right)$$

$$\left(1, a\right)$$

 $\left(0<a<1\right)$ this is a decreasing function

 *x*-axis is a horizontal asymptote

 Domain: all real numbers

 Range: $y>0$

 This is a continuous function

This graph is a reflection of the graph of $y=a^{x}$ ($a>1$) over the *y*-axis

**Transformations of Exponential Graphs**

 Example: $f\left(x\right)=2^{x}$

 $g\left(x\right)=5^{x}$

 Example: $f\left(x\right)=2^{-x}$

 $g\left(x\right)=5^{-x}$

 (reflection over *y*-axis)

 Example: $f\left(x\right)=4^{x-2}$

 (2 units to the right)

 Example: $f\left(x\right)=\frac{1}{2}\left(4\right)^{x}$

 (vertical shrink by $\frac{1}{2}$)

 Example: $f\left(x\right)=-3\left(2\right)^{x}$

 (vertical stretch by 3)

 (refection over *x*-axis)

 Example: $f\left(x\right)=4^{-x}+3$

 (reflection over *y*-axis)

 (up 3)

**Using the One-to-One Property**

 $a^{x}=a^{y}$ iff $x=y$

Example: $16=2^{x+2}$

 $2^{4}=2^{x+2}$ → $4=x+2$

 $2=x$

Example: $3^{x-1}=\frac{1}{27}$

Example: $25=125^{x}$

Example: $\left(\frac{1}{3}\right)^{x}=81$

**Natural Base *e***$e≈2.718281828…$

 $f\left(x\right)=e^{x}$ $f\left(2\right)=e^{2}≈7.389$

 $f\left(-0.3\right)=e^{-0.3}≈$

 $f\left(\frac{3}{4}\right)=e^{^{3}/\_{4}}≈$

**Graphing Natural Exponential Functions**

 $f\left(x\right)=2e^{0.16x}$

**Applications**

**Formulas for Compound Interest**

 Compounded *n* times a year $A=P\left(1+\frac{r}{n}\right)^{nt}$

 Compounded continuously $A=Pe^{rt}$

Example: A $25000 deposit is made in a trust fund that pays 8.25% interest. The money matures for 26 years. Determine the amount after 26 years if the interest is compounded:

 Quarterly

 Monthly

 Continuously

Example: Let *Q* represent the amount of Radium (half-life = 1620 years).

 The quantity of radium present after *t* years is:

 $Q=16\left(\frac{1}{2}\right)^{^{y}/\_{1620}}$

1. Sketch a graph of *Q* from $t = 0$ to $t = 5000$.
2. Determine the initial quantity of radium (when $t = 0$).
3. Determine the quantity of radium present after 1000 years.