**Algebra 2 7.4 – Radical Exponents**

Expressing rational expressions in Radical Form and Exponential Form

 Radical Form Exponential Form

 $\sqrt{25}$ $25^{^{1}/\_{2}}$

 $\sqrt[3]{27}$ $27^{^{1}/\_{3}}$

 $\sqrt[4]{16}$ $16^{^{1}/\_{4}}$

**Rational Exponents** If $\sqrt[n]{a}$ is a real number and $m$ is an integer

 $a^{^{1}/\_{n}}= \sqrt[n]{a}$ $a^{^{m}/\_{n}}= \sqrt[n]{a^{m}}= \left(\sqrt[n]{a}\right)^{m}$

Examples:

 $64^{^{1}/\_{3}}$

 $7^{^{1}/\_{2}}∙7^{^{1}/\_{2}}$

 $5^{^{1}/\_{3}}∙25^{^{1}/\_{3}}$

**Converting to and from Radical Form**

Write each expression in radical form.

 $x^{^{2}/\_{7}}$

 $y^{-0.4}$

Write each expression in exponential form

 $\sqrt[4]{c^{3}}$

 $\left(\sqrt[3]{b}\right)^{5}$

Application: The time *t* in hours needed to cook a turkey that weighs *p* pounds can be approximated by the equation:

 $t=0.89p^{0.6}$

To the nearest hundredth of an hour, how long would it take to cook a turkey that weights 13 lbs?

**Algebra 2 7.5 – Solving Radical Equations**

Radical equation – an equation that has a variable in the radicand or a variable with a rational exponent.

Examples: $3+\sqrt{x}=10$ $\left(x-2\right)^{\frac{2}{3}}=25$ $\sqrt{3}+x=10$

 radical equation radical equation NOT a radical equation

Steps for Solving Radical Equations

1. Isolate the radical
2. Eliminate the radical
3. Solve the radical-free equation
4. Check for extraneous solutions

Example: Solve: $-10+\sqrt{2x+1}$ = $-5$ check

 $\sqrt{2x+1}$ = $5$ $-10+\sqrt{2\left(12\right)+1}$ = $-5$

 $\left(\sqrt{2x+1}\right)^{2}$ = $5^{2}$ $-10+\sqrt{25}$ = $-5$

 $2x+1$ = $25$ $-10+5$ = $-5$

 $2x$ = $24$ $-5$ = $-5$

 $x$ = $12$

Example: Solve: $3\left(x+1\right)^{\frac{3}{5}}$ = $24$ check

 $\left(x+1\right)^{\frac{3}{5}}$ = $8$ $3\left(31+1\right)^{\frac{3}{5}}$ = $24$

 $\left(\left(x+1\right)^{\frac{3}{5}}\right)^{\frac{5}{3}}$ = $8^{\frac{5}{3}}$

 $x+1$ = $32$

 $x$ = $31$

Application: A spherical water tank holds 10000 ft3 of water. Find the diameter of the tank.

Use: $V=\frac{π}{6}d^{3}$

 $V$ = $\frac{π}{6}d^{3}$

 $10000$ = $\frac{π}{6}d^{3}$

 $60000$ = $πd^{3}$

 $\frac{60000}{π}$ = $d^{3}$

 $\sqrt[3]{\frac{60000}{π}}$ = $d$

 $26.73$ = $d$

**Checking for extraneous solutions**

Example: Solve: $\sqrt{x+2}-3$ = $2x$

 $\sqrt{x+2}$ = $2x+3$

 $\left(\sqrt{x+2}\right)^{2}$ = $\left(2x+3\right)^{2}$

 $x+2$ = $4x^{2}+12x+9$

 $0$ = $4x^{2}+11x+7$

 $a=4$ $b^{2}-4ac$ $x$ = $\frac{-b \pm \sqrt{b^{2}-4ac}}{2a}$

 $b=11$ $11^{2}-4\left(4\right)\left(7\right)=9$ $x$ = $\frac{-11 \pm \sqrt{9}}{2(4)}$

 $c=7$ $x$ = $\frac{-11 \pm 3}{8}$

 $x$ = $\frac{-11+ 3}{8}$ = $-1$

 $x$ = $\frac{-11- 3}{8}$ = $-\frac{14}{8}$ = $-1.75$

Check: $x=-1$ $x=-1.75$

 $\sqrt{x+2}-3$ = $2x$ $\sqrt{x+2}-3$ = $2x$

 $\sqrt{-1+2}-3$ = $2\left(-1\right)$ $\sqrt{-1.75+2}-3$ = $2\left(-1.75\right)$

 $\sqrt{1}-3$ = $-2$ $\sqrt{0.25}-3$ = $-3.5$

 $1-3$ = $-2$ $0.5-3$ = $-3.5$

 $-2$ = $-2$ $-2.5$ $\ne $ $-3.5$ This is not a solution

 It is called an *extraneous*

**Algebra 2 7.6 – Function Operations**

**Function Operations**

Addition $\left(f+g\right)\left(x\right)=f\left(x\right)+g\left(x\right)$

Subtraction $\left(f-g\right)\left(x\right)=f\left(x\right)-g\left(x\right)$

Multiplication $\left(f∙g\right)\left(x\right)=f\left(x\right)∙g\left(x\right)$

Division $\left(\frac{f}{g}\right)\left(x\right)= \frac{f\left(x\right)}{g\left(x\right)}$, $g\left(x\right)\ne 0$

Example: Let $f\left(x\right)=5x+12$ and $g\left(x\right)=3x-8$

 Find: $\left(f+g\right)\left(x\right)$ $\left(f+g\right)\left(x\right)=\left(5x+12\right)+\left(3x-8\right)$

 $ =5x+3x + 12+\left(-8\right)$

 $ =8x+4$

 Find: $\left(f-g\right)\left(x\right)$ $\left(f-g\right)\left(x\right)=\left(5x+12\right)-\left(3x-8\right)$

 $ =\left(5x+12\right)+\left(-3x+8\right)$

 $ =5x+ \left(-3x\right) + 12+8$

 $ =2x+20$

Example: Let $f\left(x\right)=x^{2}+6x$ and $g\left(x\right)=-2x+10$

 Find: $\left(f+g\right)\left(x\right)$

 Find: $\left(f-g\right)\left(x\right)$

Example: Let $f\left(x\right)=x^{2}-1$ and $g\left(x\right)=x-1$

 Find: $\left(f∙g\right)\left(x\right)$ $\left(f∙g\right)\left(x\right)=\left(x^{2}-1\right)∙\left(x-1\right)$

 $ =x^{3}-x^{2}-x+1$

 Find: $\left(\frac{f}{g}\right)\left(x\right)$ $\left(\frac{f}{g}\right)\left(x\right)=\frac{x^{2}-1}{x-1}$ $x\ne 1$

 $ =\frac{\left(x+1\right)\left(x-1\right)}{x-1}$ $x\ne 1$

 $ =x+1$ $x\ne 1$

Example: Let $f\left(x\right)=2x^{2}+8x$ and $g\left(x\right)=x+4$

 Find: $\left(f∙g\right)\left(x\right)$

 Find: $\left(\frac{f}{g}\right)\left(x\right)$

**Composition of Functions**

$$g∘f\left(x\right)=g\left(f\left(x\right)\right)$$

1. Evaluate the inner function $f\left(x\right)$ first.

2. Then use your answer as the input of the outer function $g\left(x\right)$.

Example: Let $f\left(x\right)=x^{2}$ and $g\left(x\right)=2x+3$

 Find: $g∘f\left(4\right)$ $ f\left(4\right)=4^{2}=16$

 $ g\left(16\right)=2\left(16\right)+3=35$

 $g∘f\left(4\right)=35$

 Find: $f∘g\left(4\right)$ $ g\left(4\right)=2\left(4\right)+3=11$

 $ f\left(11\right)=11^{2}=121$

 $f∘g\left(4\right)=121$

Example: Let $f\left(x\right)=2x-1$ and $g\left(x\right)=x+10$

 Find: $f∘g\left(5\right)$

 Find: $f∘g\left(-2\right)$

 Find: $g∘g\left(8\right)$

Example: Let $f\left(x\right)=x^{2}$ and $g\left(x\right)=2x+3$

 Find: $g∘f\left(x\right)$ $ f\left(x\right)=x^{2}$

 $ g\left(x^{2}\right)=2\left(x^{2}\right)+3=2x^{2}+3$

 Find: $f∘g\left(x\right)$ $ g\left(x\right)=2\left(x\right)+3=2x+3$

 $ f\left(2x+3\right)=\left(2x+3\right)^{2}=4x^{2}+12x+9$

Example: Let $f\left(x\right)=2x-1$ and $g\left(x\right)=x+10$

 Find: $f∘g\left(x\right)$

 Find: $g∘f\left(x\right)$

**Algebra 2 7.7 – Inverse Relations and Functions**

An INVERSE RELATION (or FUNCTION) “undoes” the original relation.

We will use the symbol $f^{-1}\left(x\right)$ to represent the inverse of $f\left(x\right)$.

We can find the inverse by switching the $x\leftrightarrow y$ coordinates.

 Relation Inverse of the Relation

|  |  |
| --- | --- |
| *x* | *y* |
| -3 | 5 |
| 2 | 0 |
| 1 | 3 |

|  |  |
| --- | --- |
| *x* | *y* |
|  |  |
|  |  |
|  |  |

Finding the inverse of an equation:

$$f\left(x\right)=x^{2}-2$$

Find the inverse of $f\left(x\right)=4x-7$

Find the inverse of $f\left(x\right)=\sqrt{x+9}$

 $f\left(x\right)$ $f^{-1}\left(x\right)$

 Domain Domain = Range of $f\left(x\right)$

 Range Range = Domain of $f\left(x\right)$

Composition of Inverse Functions

 $\left(f^{-1}∘f\right)\left(x\right)=x$ and $\left(f∘f^{-1}\right)\left(x\right)=x$

Example: Show that the given functions are inverses of each other.

 $f\left(x\right)=\frac{1}{2}x+5$

 $f^{-1}\left(x\right)=2x-10$

Example:

a. Find the domain and range of $f\left(x\right)=\sqrt{2x+2}$

 Domain:

 Range:

b. Find the inverse of $f\left(x\right)=\sqrt{2x+2}$

c. Find the domain and range of $f^{-1}\left(x\right)$

 Domain:

 Range:

**Algebra 2 7.8 – Graphing Radical Functions**

Translating square root functions vertically (*k*)

 Graph: $y=\sqrt{x}$

 $y=\sqrt{x}+5$

 $y=\sqrt{x}-3$

Translating square root functions horizontally (*h*)

 Graph: $y=\sqrt{x}$

 $y=\sqrt{x+3}$

 $y=\sqrt{x-5}$

Graphing square root functions (*a*)

 Graph: $y=\sqrt{x}$

 $y=-\sqrt{x}$

 $y=2\sqrt{x}$

 $y=-3\sqrt{x}$

Example:

 Graph: $y=3 \sqrt{x+2}-1$

 (*a* = 3) (left 2 and down 1)

Example:

 Graph: $y=2 \sqrt{x-4}+3$

Example:

 Graph: $y=- \sqrt{x}+4$

Translating cube root functions vertically (*k*)

 Graph: $y=\sqrt[3]{x}$

 $y=\sqrt[3]{x}+4$

 $y=\sqrt[3]{x}-2$

Translating cube root functions horizontally (*h*)

 Graph: $y=\sqrt[3]{x}$

 $y=\sqrt[3]{x+1}$

 $y=\sqrt[3]{x-4}$

Graphing cube root functions (*a*)

 Graph: $y=\sqrt[3]{x}$

 $y=-\sqrt[3]{x}$

 $y=2 \sqrt[3]{x}$

 $y=3 \sqrt[3]{x}$

Example:

 Graph: $y=2 \sqrt[3]{x-1}+3$

 (*a* = 2) (right 1 and up 3)

Example:

 Graph: : $y= \sqrt[3]{x+2}-5$

Example:

 Graph: $y=- \sqrt[3]{x+4}$