**8.2 Operations with Matrices**

**Representation of Matrices**

1. A matrix can be denoted by an uppercase letter: ex: *A*, *B*, *C*, etc.
2. A matrix can be denoted by a representative element enclosed in brackets: $\left[a\_{ij}\right]$, $\left[b\_{ij}\right]$…
3. A matrix can be denoted as a rectangular array of numbers

 $A= \left[a\_{mn}\right]= \left[\begin{matrix}\begin{matrix}a\_{11}&a\_{12}\\a\_{21}&a\_{22}\end{matrix}&\begin{matrix}\cdots &a\_{1n}\\\cdots &a\_{2n}\end{matrix}\\\begin{matrix}\vdots &\vdots \\a\_{m1}&a\_{m1}\end{matrix}&\begin{matrix}&\vdots \\\cdots &a\_{mn}\end{matrix}\end{matrix}\right]$

**Equal Matrices – two matrices are equal if they have the same order (dimensions) and their corresponding entries are equal.**

Ex: Solve for $a\_{11}$, $a\_{12}$, $a\_{21}$ and $a\_{22}$

 $\left[ \begin{matrix}a\_{11}&a\_{12}\\a\_{21}&a\_{22}\end{matrix} \right]= \left[ \begin{matrix}-3&-2\\ 7& 13\end{matrix} \right]$

Ex: Solve for each variable.

 $\left[ \begin{matrix}2x&7\\ 3&y\end{matrix} \right]= \left[ \begin{matrix}16& 7\\ 3&-4\end{matrix} \right]$

**Addition of Matrices:** two matrices may be added if they have the same order. Add their corresponding entries.

Ex: $\left[ \begin{matrix}4&-1\\2&-3\end{matrix} \right] + \left[ \begin{matrix}2&-1\\0& 6\end{matrix} \right]$

Ex: $\left[ \begin{matrix}1&5&-3\\0&2& 7\end{matrix}\right] + \left[ \begin{matrix} 0&0&0\\-3&4&1\end{matrix}\right]$

Ex: $\left[ \begin{matrix}1&2&3\\0&4&5 \\1&6&7\end{matrix}\right] + \left[ \begin{matrix}-3&1\\ 4&7\\-5&0\end{matrix} \right]$

**Scalar Multiplication -** a matrix can be multiplied by a scalar

 (multiply each entry by the scalar)

 $A= \left[\begin{matrix} 4&-1\\ 0& 4\\-3& 8\end{matrix} \right]$ $B= \left[\begin{matrix} 0&4\\-1&3\\ 1&7\end{matrix} \right]$

Ex: $-3A=$

Ex: $2A+4B=$

Ex: $4A-B=$

Ex: $A=\left[\begin{matrix} 4\\-1\\ 2\end{matrix}\right]$ $B=\left[\begin{matrix}0\\2\\0\end{matrix}\right]$ $C=\left[\begin{matrix}-3\\-1\\-2\end{matrix}\right]$ $D=\left[\begin{matrix}-3\\ 5\\ 2\end{matrix}\right]$

 Find: $A+B+C+D$

Ex: $2\left(\left[\begin{matrix} 1&3\\-2&2\end{matrix}\right]+\left[\begin{matrix}-4&0\\-3&1\end{matrix}\right]\right)$

Ex: $3\left[\begin{matrix} 4&1\\-2&6\end{matrix}\right]-7\left[\begin{matrix}0& 3\\1&-2\end{matrix}\right]$

**Matrices on TI-84 calculators**

**Solving a Matrix Equation** $A=\left[ \begin{matrix}1&-2\\0& 3\end{matrix} \right]$ $B=\left[ \begin{matrix}-3&4\\2&1\end{matrix} \right]$

Solve for $X$ in the equation $3X+A=B$

**Matrix Multiplication:** Matrices *A* and *B* can be multiplied if the number of columns in matrix *A* matches the number of rows in matrix *B*.

 $A\_{mn} ∙ B\_{np}$ the result is a $m×p$ matrix

$A=\left[ \begin{matrix}2&-1& 0\\5&-2&-6\end{matrix} \right]$ $B=\left[\begin{matrix}-7&1\\ 0&5\\-2&1\end{matrix} \right]$

Find $AB$

Find $BA$

more multiplication examples

Ex: $\left[ \begin{matrix}0&1\\7&2\end{matrix} \right]∙\left[ \begin{matrix}0&3&-2\\2&1&4\end{matrix} \right]$

Ex: $\left[ \begin{matrix} 2&1\\-4&0\\ 1&3\end{matrix} \right]∙\left[ \begin{matrix} 3\\-1\end{matrix} \right]$

Ex: $\left[ \begin{matrix} 0&2&4\\-1&1&2\end{matrix} \right]∙\left[ \begin{matrix}2&-4&5\\0& 9&3\end{matrix} \right]$

Ex: $\left[ \begin{matrix}\begin{matrix}0&1\end{matrix}&\begin{matrix}5&-3 \end{matrix}\end{matrix}\right]∙\left[ \begin{matrix}\begin{matrix} 3\\ 5\end{matrix}\\\begin{matrix}-1\\ 2\end{matrix}\end{matrix} \right]$

Ex: $\left[ \begin{matrix}\begin{matrix} 0\\ 1\end{matrix}\\\begin{matrix} 4\\-2\end{matrix}\end{matrix} \right]∙\left[ \begin{matrix}\begin{matrix}-1&2\end{matrix}&\begin{matrix}0&3 \end{matrix}\end{matrix}\right]$

**Identity Matrix (must be a square matrix)**

 $I=\left[\begin{matrix}\begin{matrix}\begin{matrix} 1&0&0\end{matrix}\\\begin{matrix} 0&1&0\end{matrix}\\\begin{matrix} 0&0&1\end{matrix}\end{matrix}&\begin{matrix}\cdots \\\cdots \\\cdots \end{matrix}&\begin{matrix}0\\0\\0\end{matrix}\\ \begin{matrix}\vdots & \vdots & \vdots \end{matrix}&&\vdots \\\begin{matrix} 0&0&0\end{matrix}&\cdots &1\end{matrix} \right]$

Ex: $\left[\begin{matrix} 3&1\\-2&5\\ 0&3\end{matrix}\right]∙\left[ \begin{matrix}1&0\\0&1\end{matrix} \right]$

Write the system of linear equations as a matrix equation ($AX=B$)

Use Gauss-Jordan Elimination on the augmented matrix $\left[A:B\right]$ to solve for matrix $X$.

 $\left\{\begin{matrix} x\_{1}+ x\_{2}-3x\_{3}=-1\\-x\_{1}+2x\_{2} = 1\\ x\_{1}- x\_{2}+ x\_{3}=-1\end{matrix}\right.$

An electronics manufacturer produces 3 models of TVs which are shipped to two warehouses. The number of units of model *i* that are shipped to location *j* are represented by the matrix $\left[a\_{ij}\right]$.

 $A= \left[\begin{matrix}5000& 4000\\6000&10000\\8000& 5000\end{matrix}\right]$ The prices per unit are represented by the matrix:

 $B= \left[\begin{matrix}\$699.95&899.95&1099.95\end{matrix}\right]$

 Compute *BA* and interpret the results.