# Algebra 2 Sequences Name:

**Arithmetic Sequence – the difference between consecutive terms is a constant (*d*)**

 ***d* is called the common difference.**

Identifying an Arithmetic Sequence:

 ex: $12, 16, 20, 24, …$

 $16-12=4$ yes, the sequence is arithmetic $d=4$

 $20-16=4$

 $24-20=4$

 ex: $87, 80, 73, 66, … $

 ex: $1, 3, 6, 10, …$

**Arithmetic Sequence Formulas**

 Recursive Explicit

 $\left\{\begin{matrix}a\_{1}=a given value\\a\_{n}=a\_{n-1}+d \end{matrix}\right.$ $a\_{n}=a\_{1}+\left(n-1\right)d$

$$a\_{1}=first term$$

$$a\_{n}=nth term$$

$$a\_{n-1}=the previous term$$

$$d=common differnce$$

Find the first five terms of the arithmetic sequence:

 ex: $\left\{\begin{matrix}a\_{1}=20 \\a\_{n}=a\_{n-1}+8\end{matrix}\right.$ $a\_{1}=20$

 $a\_{2}=20+8=28$

 Recursive formula $a\_{3}=28+8=36$

 $a\_{4}=36+8=44$

 $a\_{5}=44+8=52$ $20, 28, 36, 44, 52…$

 ex: $a\_{n}=16+\left(n-1\right)3$ $a\_{1}=16+\left(1-1\right)3=16$

 $a\_{2}=16+\left(2-1\right)3=19$

 Explicit formula $a\_{3}=16+\left(3-1\right)3=22$

 $a\_{4}=16+\left(4-1\right)3=25$

 $a\_{5}=16+\left(5-1\right)3=28$ $16, 19, 22, 25, 28, …$

Find the 20th term of each of the above sequences:

 $20, 28, 36, 44, 52…$

 $\left\{\begin{matrix}a\_{1}=20 \\a\_{n}=a\_{n-1}+8\end{matrix}\right.$

 $16, 19, 22, 25, 28, …$

 $a\_{n}=16+\left(n-1\right)3$

Example: Find the 75th term of the arithmetic sequence: $12, 4, -4, -12, …$

APPLICATION: *Suppose you have already saved $75 toward the purchase of a new phone. You plan to save $12 each week from your part-time job. What is the amount you will have saved after 26 weeks?*

 Week Savings $a\_{n}=a\_{1}+\left(n-1\right)d$

 1 $87

 2 $99 $a\_{n}=87+\left(n-1\right)12$

 3 $111

 4 $123 $a\_{26}=87+\left(26-1\right)12=\$387$

**Arithmetic Mean – the arithmetic mean of any two numbers is the average of the two numbers.**

 Arithmetic Mean = $\frac{sum of the two numbers}{2}$

We can use the arithmetic mean to find a missing term from an arithmetic sequence.

 ex: Find the missing term of the arithmetic sequence

 $…, 84, , 110, …$ arithmetic mean = $\frac{84+110}{2}=97$

 $…, 84, 97, 110, …$

 ex: Find the arithmetic mean $a\_{n}$ of the given terms.

 $a\_{n-1}=0.3$ $a\_{n+1}=1.9$

 ex: find the missing terms of the arithmetic sequence.

 $…, 12, , , , 28, … $

**Geometric Sequence – the ratio between consecutive terms is a constant (*r*)**

 ***r* is called the common ratio.**

Identifying an Geometric Sequence:

 ex: $1, -6, 36, -216, …$

 $-6÷1=-6$ yes, the sequence is geometric $r=-6$

 $36÷-6=-6$

 $-216÷36=-6$

 ex: $2, 4, 6, 8, … $

 ex: $100, 90, 81, 72.9, …$

**Geometric Sequence Formulas**

 Recursive Explicit

 $\left\{\begin{matrix}a\_{1}=a given value\\a\_{n}=a\_{n-1}∙r \end{matrix}\right.$ $a\_{n}=a\_{1}∙r^{n-1}$

$$a\_{1}=first term$$

$$a\_{n}=nth term$$

$$a\_{n-1}=the previous term$$

$$r=common ratio$$

 ex: Write a recursive and explicit formula for the sequence: $2, -10, 50, -250, …$

 $-10÷2=-5$ The sequence is geometric $r=-5$

 $50÷-10=-5$ $a\_{1}=2$

 $-250÷50=-5$

 Recursive formula Explicit Formula

 $\left\{\begin{matrix}a\_{1}=2 \\a\_{n}=-5∙a\_{n-1}\end{matrix}\right.$ $a\_{n}=2∙\left(-5\right)^{n-1}$

 Find the first 5 terms find the 10th term

ex: Write a recursive and explicit formula for the geometric sequence: $0.7, 0.07, 0.007, …$

 Recursive formula Explicit Formula

APPLICATION: *Suppose you want to enlarge a photo to 120% of its original size. The photo has a length of 10 cm. Find the length of the photo after 5 enlargements of 120%.*

 Enlargement Length $a\_{n}=a\_{1}∙r^{n-1}$

 1 $1.20\left(10\right)=12$

 2 $1.20\left(12\right)=14.4$ $a\_{5}=12∙\left(1.20\right)^{5-1}$

 3 $1.20\left(14.4\right)=120$

 $a\_{5}≈24.9 cm$

**Geometric Mean – the geometric mean of any two numbers is positive square root of the product of the two numbers.**

 Geometric Mean = $\sqrt{product of the two numbers}$

Find the geometric mean of 2 and 50.

 Geometric mean = $\sqrt{2∙50}=\sqrt{100}=10$

We can use the geometric mean to find a missing term from a geometric sequence.

 ex: Find the missing term of the geometric sequence

 $…, 5, , 911.25, …$ geometric mean = $\sqrt{5\left(911.25\right)}=67.5$

 $…, 5, 67.5, 911.25, …$

 ex: Find the missing terms of the geometric sequence:

 $2.5, , , , 202.5$